Lab 17

Purpose

To explore a variety curves in the plane.

17.1 Families of Curves

Whereas most calculus textbooks discuss graphs of functions on an individual basis, it is most instructive to experiment with functions which depend on one parameter. Thus, for example, instead of graphing $f(x) = x^2 + 5$ and $f(x) = x^2 - 5$ as separate functions, you could graph instead $f(x) = x^2 + c$, where c is varied between -5 and 5. What you would find in this case is that the set of graphs would consist of a family of parabolas of identical shapes, but different $y-$intercepts. Such a family of curves is called a one-parameter family of functions and the quantity c is called the parameter. You would conclude that varying the parameter c just results in a traslation or shift of the graph along the $y-$axis. In a similar fashion, the set of curves $f(x) = cx^2$ consists of a family of parabolas passing through the origin, but with different apertures (concavity). When the value of c is positive, the parabolas are concave up and when c is negative, they are concave down. when $c = 0$, the parabolas degenerate into a straight line.

Instructions

To investigate a one-parameter family of functions such as $y = x^2 + c$, use the following guidelines:

- let $f(x)$ be the function with $c = 0$ and let $g(x)$ be the function with the arbitrary parameter. You can then plot both functions in the same graph and describe the effect of varying the value of c.
- Make sure that you experiment with a good range of values including some negative numbers if appropriate.
- For each set of graphs, select by hand a good range for the variables in the plot box that exhibits the main features of the functions and thereafter keep those fixed. If you let MathCAD select the ranges automatically, it might be difficult to observe changes in the size and concavity of the family of curves.
- Beware of division by 0. This will result in a "singularity" error message.
- Describe qualitatively all your observations.

Exercises

Use MathCAD to investigate the result of varying the parameter for the following functions.

▷ **Exercise 17.1** $f(x) = x^3 + c$

▷ **Exercise 17.2** $f(x) = cx^3$

▷ **Exercise 17.3** $f(x) = x^3 + cx$

▷ **Exercise 17.4** $f(x) = x^4 + cx$

▷ **Exercise 17.5** $f(x) = \dfrac{1}{x + c}$

▷ **Exercise 17.6** $f(x) = \dfrac{1}{x^2 + c}$

▷ **Exercise 17.7** $f(x) = \dfrac{x}{x^2 + c}$

▷ **Exercise 17.8** $f(x) = \dfrac{x^2}{x^2 + c}$

▷ **Exercise 17.9** $f(x) = x + \dfrac{c}{x}$

▷ **Exercise 17.10** $f(x) = x^2 + \dfrac{c}{x}$