Lab 21

Purpose

To explore the dynamics of a simple harmonic oscillator (SHO), using a simple pendulum.

21.1 The Simple Pendulum

Simple harmonic motion occurs when an obect is subjected to a force that is proportional to the negative of the displacement. This means that as the displacement increases, so does the force, except that the direction of the force is opposite to that of the displacement.
A common example of a simple harmonic oscillator is the simple pendulum. Consider an object of mass m attached to one end of a string of length l. The other end of the string is attached to a rigid horizontal bar. The mass is displaced from equilibrium by a small amount s_0 and then released. Assume that the angle that the string makes with the vertical at the initial position is of measure θ_0. As the pendulum is released, the action of gravity sets the pendulum in motion along an arc of a circle of radius l. The linear displacement of the mass is given by the arclength formula

$$s(t) = l\theta(t), \tag{21.1}$$

where θ is the angular displacement at time t.

Only the component of gravity tangential to the direction of motion, contributes to the restoring force acting on the pendulum. This tangential component F_T is given by [see figure above]

$$F_T = -mg\sin\theta = -mg\sin\frac{s}{l}. \tag{21.2}$$

The equation of motion is given by applying Newton's second law

$$m\frac{d^2s}{dt^2} = F_T \tag{21.3}$$

$$m\frac{d^2s}{dt^2} = -mg\sin\frac{s}{l}. \tag{21.4}$$

The mass m cancels and the last equation becomes

$$\frac{d^2s}{dt^2} = -g\sin\frac{s}{l}. \tag{21.5}$$

The appearance of the $\sin(s/l)$ term in (21.5) makes the equation non-linear and very difficult to solve. On the other hand, if we assume that $\theta = s/l$ is sufficiently small, we can use the approximation

$$\sin\theta \approx \theta = \frac{s}{l}.$$

Equation (21.5) then reduces to

$$\frac{d^2 s}{dt^2} = -\omega^2 s, \tag{21.6}$$

where,

$$\omega^2 = \frac{g}{l}. \tag{21.7}$$

We shall refer to equation (21.6) as the undamped SHO differential equation. To solve this equation we must find a function $s(t)$, whose second derivative is a negative multiple of the function. The solution is found by inspection, since the only functions with this property are the sine and the cosine functions. In fact, it is not difficult to verify that the solution is of the form

$$s = A\cos\omega t + B\sin\omega t, \tag{21.8}$$

where A and B are arbitrary constants.

▷ **Exercise 21.1** Verify that (21.8) is a solution of (21.6).

Assume that the oscillator is set into motion by displacing the mass s_0 units away from the equilibrium position and then releasing it. The initial conditions are then

$$s(0) = s_0, \ \frac{ds}{dt}(0) = 0 \tag{21.9}$$

The initial conditions will be satisfied if we choose $A = s_0$ and $B = 0$. The desired solution of the differential equation is therefore

$$s(t) = s_0 \cos\omega t. \tag{21.10}$$

The quantity s_0 is called the *amplitude* and ω is called the *natural frequency.* The *period* T of the oscillation is given by

$$T = \frac{2\pi}{\omega} \tag{21.11}$$

The period (measured in seconds) represents the time it takes for the pendulum to complete a complete cycle. The amplitude is independent of the mass. It is a remarkable fact, that as long as the amplitude of oscillation is not too large, the period is also independent of the amplitude.

21.2 Pendulum Experiment

Materials

- MBL equipment with distance probe.
- 1 meter rod with clamp.
- 1 metal bar clamp
- 1 ball attached to a string

Procedure

- Clamp a the 1 meter rod to the edge of a laboratory table.
- Clamp the metal bar horizontally, pointing towards the table and as high as possible along the rod.
- Hang the pendulum using the miniclamp with the metal knob at the end of the metal bar clamp.
- Place the MBL distance probe vertically over the table near the pendulum. Adjust the length of the pendulum so that the center of the spherical mass is aligned with the center of the sonar emitter of the distance probe.
- Move the probe about 60 cm. away from the pendulum. Displace the pendulum about 3 cm. from equilibrium, being extremely careful to insure the direction motion is aligned with the probe.
- Do a few trial runs until you get a smooth graph.
- Record 6 to 8 cycles of the motion of the pendulum. Save the data into a spreadsheet.

Questions

- Use the data to compute the average period of the pendulum.
- Measure the length of the pendulum with the maximum accuracy possible. The effective length of the pendulum is the length of the string plus the radius of the ball. The diameter of the ball can be measured with a caliper. Each member of the team should take an independent measurement and the results averaged.
- Using (21.7) calculate the acceleration g due to gravity.
- Plot on the same graph, the velocity and the position versus time. Discuss the relation between the zeroes, and the extrema of the two graphs. Explain this relation in terms of the physics of the oscillation.
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  - Plot the velocity versus the position. What kind of graph did you get?
  - Give a parametric equation for this graph.
  - Eliminate the time parameter from the equation and thus write the equation in "Cartesian" form.