Lab 18

Purpose

To implement the trapezoidal rule in MathCAD in the evaluation of integrals over probability densities.

Files

RIEMM.MCD

18.1 Probability Distributions

Suppose an experiment, or a sequence of measurements is performed whose outcome is any real value X in some (possibly infinite) interval $[A, B]$. For example, the values of X may refer to the results of measuring the heights of individuals, or the lifetime of collection of battery cells. Since the outcome can assume values from a continuous set, we refer to X as a *continous random variable*. It may be possible to find a function $f(x)$ on $[A, B]$ such that the probability $P(a \leq X \leq b)$ that the value of X be between a and b is given by

$$P(a \leq X \leq b) = \int_a^b f(x)dx. \tag{18.1}$$

The function $f(x)$ is required to satisfy the following two conditions[1]:

$$1)\ f(x) \geq 0, \quad \forall x. \tag{18.2}$$

$$2)\ \int_A^B f(x)dx = 1. \tag{18.3}$$

The function $f(x)$ is then called a *probability density* or a *probability distribution*.

The *expectation* or *mean* value $E(X)$ of the variable X is defined by

$$E(X) = \int_A^B xf(x)dx. \tag{18.4}$$

[1] See: Goldstein L, Lay D and Schneider D. *Calculus and its Applications*. Prentice Hall, 1990

18.2 Probability and Numerical Integration

Load the file **RIEMM.MCD** into MathCAD and modify it to implement the trapezoidal rule. Using MathCAD to evaluate all the integrals, solve the following problems.

▷ **Exercise 18.1** Verify that the following are probability densities in the given intervals:
a) $f(x) = 4e^{-4x}$ on $[0, \infty)$.
b) $f(x) = 20x^3(1-x)$ on $[0, 1]$.

▷ **Exercise 18.2** Find the value of c so that function $f(x) = cx^2e^{-x}$ represents a probability density on $[0, \infty)$.

▷ **Exercise 18.3**
a) The probability of finding an electron in a s-state in a hydrogen atom at a distance r from the center of the atom is given by a function of the form [see equation (16.3).]

$$f(r) = kr^2(2-r)^2e^{-r}.$$

Find out what should the value of k be.
b) What is the expectation value of r in this case? What is the physical meaning of this expectation value?

▷ **Exercise 18.4**
a) Find the value of k so that $f(x) = ke^{-\frac{1}{2}x^2}$ represents a probability density on $(-\infty, \infty)$
b) Compute $1/2k^2$. Use the result to write k in closed form.
c) What is the expectation value of x for this distribution?
d) Evaluate the value of erf(1) to 3 decimals, where

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

▷ **Exercise 18.5** The gestation period of pregnant females of a certain species is represented by a random variable T with probability density

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}[\frac{t-\mu}{\sigma}]^2},$$

where σ is 6 months and μ is $\frac{1}{2}$ month. What is the probability that a birth will take place between 6 and 7 months.