Nonlinear Boundary Value Problems

Organizer: Anna Capietto, University of Torino, Italy

Critical and subcritical nonlinear Schrödinger equation with magnetic field

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We consider the nonlinear stationary Schrödinger equation in a magnetic field $\hat{H}\psi = f(\psi)$, where $\hat{H} = \frac{1}{2m}\left(-i\hbar\nabla - \frac{e}{c}A\right)^2 + eV$. V is the scalar potential and A is the vector potential. We study the existence and nonexistence of solutions for the equation under different assumptions on the potentials and on the nonlinear part f. We consider both the critical case $f(x) = |x|^{2^*-2}x$ and the subcritical case $|f(x)| < |x|^p$, $p < 2^* - 1$. The results are obtained by variational methods; in particular the solutions are obtained either as constrained minima or as mountain pass points.

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Multiple positive solutions for quasilinear elliptic boundary value problems

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R. Shivaji

We will discuss multiplicity of positive solutions for a class of quasilinear elliptic boundary value problem. We prove our result by using the method of sub and super solutions.

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On the existence of solutions and their long time behavior of a nonlocal thermistor inequality

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We study an obstacle problem which models the behavior of certain micromachined microsensor devices:

$$r\frac{du}{dt} - \Delta u + \eta\int_\Omega G(x,y)u(y,t)dy + \gamma u^4$$
$$\geq \nabla[\sigma(u)\phi\nabla\phi],$$
$$-\nabla[\sigma(u)\nabla\phi] = 0.$$

Here the unknown functions u and ϕ denotes the distributions of the temperature and the electrical potential in the electrical devices respectively. Let Ω be a domain in R^3. Its boundary $\partial\Omega$ is divided into three parts, i.e., $\partial\Omega = \Gamma_0 \cup \Gamma_1 \cup \Gamma_N$. On the boundary, the temperature u satisfies a homogenous Dirichlet boundary condition. While the potential ϕ satisfies $\phi|_{\Gamma_0} = \phi_0(x,t)$, $\frac{\partial\phi}{\partial n}|_{\Gamma_N} = 0$ and $\phi|_{\Gamma_1} = \xi(t)$. Here $\xi(t)$ is an unknown constant for each t. But the total current $I(t)$ through Γ_1 is given for each time t. Thus another nonlocal boundary condition is given by

$$I(t) = \int_{\Gamma_1}\sigma(u)\frac{d\phi}{dn}ds.$$

We first consider the initial value case, i.e., $u(x,0) = u_0(x)$ with $u_0(x)$ a known function. The existence of a unique solution is established by a penalized method. Next, if $\phi_0(x,t)$ and $I(t)$ are time periodic functions with period T, by setting up a Poicaré map and applying Schauder's fixed point theorem we show that there exists a time periodic solution such that $u(x,t+T) = u(x,t)$. Finally the long time behavior of the solutions is studied. Actually it is characterized by a uniform attractor. The upper bound of the dimensions of the attractors are also investigated.

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Some multi-point boundary value problems containing the operator $-(\phi(u'))'$

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In this talk we will consider some result for problems of the form

$$(\phi(u'))' = f(t,u,u'), \quad t \in (a,b),$$

under the multi-point boundary conditions

$$u(a) = 0, \quad u'(\eta) = u'(b),$$

and

$$u'(a) = 0, \quad u(\eta) = u(b),$$

where $\eta \in (a,b)$ is given. We will consider the case when problem (P) is at resonance. Three-point boundary value problems at resonance have been studied in several papers, we present some new result as well as generalizations to the general operator of other results valid for particular forms of the operator $-(\phi(u'))'$.

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Boundary value problems on sequence spaces

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In this paper we study the solvability and qualitative properties of solutions of nonlinear operator equations of the form

$$Lx = F(u,x)$$

subject to

$$H(u,x) = 0$$

where x belongs to a sequence space,u is a parameter, L is a linear map from the sequence space into itself, F and H are smooth nonlinear maps, the range of F is contained in the sequence space and the range of H is contained in some n-dimensional Euclidean space. Connections with differential equations will be established.

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Nonlinear boundary value problems of the Calculus of Variations

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We consider various nonlinear boundary value problems arising in the extremal theory for the functional

$$I(x) = \int_a^b L(t,x,x')\,dt,$$

where L is sufficiently smooth. The free end point problem, the basic problem of the calculus of variations, the Bolza problem and others are considered. We are interested mainly in the existence of extremals (solutions of the Euler equation $L_x = \frac{d}{dt}L_{x'}$ together with related boundary conditions) and properties of extremals. First, we discuss the method of upper and lower solutions (functions) and its variational meaning. Second, we derive several the Bernstein - Nagumo type conditions, which are formulated directly in terms of the Lagrangian L. We show the role of those conditions in the theory of coercive, noncoercive (slow-growth) variational problems and discuss also the so called problem of regularity of solutions in the calculus of variations. Third, we consider properties of extremals, which are expressed in terms of the linearized equation (the Jacobi equation) and discuss several problems arising in the theory of the second variation for the functional $I(x)$.

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Existence and uniqueness for a class of quasilinear elliptic boundary value problems

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We prove existence and uniqueness of positive solutions for the boundary value problem $(r^{N-1}\phi(u'))' = -\lambda r^{N-1}f(u),\ u'(0) = u(1) = 0$, where $\phi(x) = |x|^{p-2}x$, $\frac{f(x)}{x^{p-1}}$ may not be decreasing on $(0,\infty)$, and λ is a large parameter.

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Some multiplicity results for polyharmonic elliptic problems with broken of symmetry

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By means of a perturbation argument devised by P. Bolle we investigate the existence of infinitely many solutions for the problem $(-\Delta)^K u = |u|^{\sigma-2}u + \varphi$ in Ω with nonhomogeneous Dirichlet boundary conditions $\left(\frac{\partial}{\partial\nu}\right)^j u\Big|_{\partial\Omega} = \phi_j \quad j = 0,\ldots,K-1$, provided that suitable growth restrictions on σ are assumed.

Moreover, when σ reaches the critical growth and $\phi_j = 0$, we show the existence of multiple solutions when the domain and the nonhomogenous term are invariant with respect to some group of symmetries.

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Periodic solutions to some N body problems

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A remarkable new solution of the 3-body problem was recently discovered by Chenciner and Mongomery, using some symmtery argument. We shall show how many non collision periodic solutions can be constructed exploiting symmetries of the systems.

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On the existence of periodic solutions to second order ordinary differential equations

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We consider second order differential equations and systems of the form

$$u'' + g(u) = p(t),$$

where g is a continuous function and $p(t+T) = p(t)$. In the scalar case we assume $g(s)s \geq 0$ for $|s|$ sufficiently large. We provide sufficient conditions for the existence of T-periodic solutions.

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