

Statistical Description of the Dynamics of Large and Disordered Systems

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Normal modes on average in stochastic systems

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In studying non linear Hamiltonian systems, the main attention has been generally posed on the range of validity of the near integrable behaviour, as outlined by the KAM and Nekhoroshev theories. Our approach, tested mainly on Fermi-Pasta-Ulam model but also on Lennard-Jones and Toda systems, consists in looking for the survival of some other kind of near integrable behaviour in the regime of motion where KAM theorem is no more applicable, i.e. above the so called strong stochastically threshold. We explore the persistence of a pseudo-harmonic spectrum (not connected to a perturbative approach), and the properties of some geometrical quantities as the Frenet-Serret curvatures, and their relations to dynamical properties. The stability of our results in thermodynamic limit has been numerically checked, always starting from generical initial conditions. The type of phase-space chaoticity in presence of pseudo-harmonicity has been studied by usual spectral methods on physically meaningful time series.

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Stages of energy transfer in the Fermi-Pasta-Ulam system

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The (alpha) Fermi-Pasta-Ulam model of weakly coupled nonlinear oscillators was originally introduced to study the dynamical relaxation of a system toward thermodynamic equilibrium. The original simulations fell in a regime which famously failed to exhibit this convergence. We revisit the original question for larger systems at energies sufficiently large to allow the system to exhibit relaxation toward thermal equilibrium. Direct numerical simulations reveal several interesting stages in this process as the energy evolves from a large-scale excitation toward

equipartition among all modes. We focus on characterizing these intermediate stages of energy transfer in physical terms.

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Statistically relevant conserved quantities for the truncated Burgers-Hopf equation

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The importance of the cubic conserved quantity (Hamiltonian) for the equilibrium statistical mechanics will be discussed. By direct numerical simulation, corrections to energy equipartition will be computed that are due to the atypical values of Hamiltonian, which is different from the energy. Statistical-mechanical arguments will be presented explaining that for randomly-chosen initial conditions, these corrections are negligible. An alternative computation of the corrections to energy equipartition via a purely statistical-mechanical Monte-Carlo simulation will also be discussed.

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Application of weak turbulence theory to Fermi-Pasta-Ulam system

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Weak turbulence theory was developed primarily to predict the properties of the stationary energy spectrum for weakly interacting turbulent waves in fluids, but the mathematical formalism can be applied to general weakly nonlinear Hamiltonian systems with diffuse coupling among modes. We adapt weak turbulence theory to obtain dynamical scaling predictions for the shape of the energy spectrum in the Fermi-Pasta-Ulam model at various intermediate stages of its evolution, and the time scales required to achieve these stages. To do this, we introduce some generalizations to weak turbulence theory which have broader application.

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Remarkable statistical behavior for truncated Burgers-Hopf Dynamics

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The Fourier-Galerkin spectral truncations of the inviscid Burgers-Hopf equation are introduced as simple one-dimensional models with a well-defined mathematical structure, intrinsic chaos and scale separation. Energy-based statistical mechanical predictions of energy equipartition and a correlation scaling theory that involves eddy viscosity will be presented. Computational results will be shown that confirm the predictions of these theories, and also shed further light on global phase-space properties of the Burgers-Hopf system, such as a number of exactly-obtainable invariant subspaces.

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