

Topological Methods for Boundary Value Problems

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Singular nonlinear boundary value problems with multiple positive solutions

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We extend recent results of Henderson and Thompson, Baxley and Haywood, and Graef, Qian, and Yang, which provided conditions on the nonlinear function $f(y)$ in order that the boundary value problem $(-1)^n y^{(2n)} = f(y)$, $y^{(2k)}(0) = 0$, $y^{(2k)}(1) = 0$, for $k = 0, \dots, n-1$ have multiple symmetric positive solutions. Since such solutions must satisfy $y^{(2k+1)}(1/2) = 0$, for $k = 0, \dots, n-1$, we consider the more general problem $L(y) = f(y)$, where L is the n th iterate of the Sturm-Liouville operator $-\frac{1}{w}(py')'$, with boundary conditions $y^{(2k)}(0) = 0$, $y^{(2k+1)}(b) = 0$, $b > 0$, for $k = 0, \dots, n-1$. The conditions we obtain allow singular behavior in the operator L at $x = 0$. In the case $w \equiv p \equiv 1$, $b = 1/2$, our conclusions reduce to the earlier results mentioned above. Previous work using the Leggett-Williams fixed point theorem or a fixed point theorem of Krasnosel'skii has used properties of relevant Green's functions. Here we use a refined version of the same theorem of Krasnosel'skii, but need no use of Green's functions to obtain the necessary estimates.

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Existence theorems for weakly inward semilinear operators

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We obtain existence theorems for semilinear equations of the form $Lx = Nx$, where the operators L and N satisfy a weakly inward condition and are such that $L - N$ is a proper. In particular, results involving positive and multiple solutions are proved.

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Kelvin-Helmholtz instability waves and upstream propagating acoustic waves in super-

sonic multiple jets

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Jet aircraft were introduced right after that the Second World War. Shortly after that, jet noise prediction and reduction became an important research topic. Because of the need for large thrust, many high performance military aircraft are propelled by two or more jet engines housed close to each other. Three Physics Laws, Conservation of Mass, Momentum and Energy, in differentiation form are employed to formulate the Kelvin-Helmholtz Instability problem of supersonic triple jets. The general solution of the system about pressure of the jets and the dispersion relation for instability waves about jet noise are derived.

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Spectrum of positively homogeneous operators and applications

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The spectral theory for nonlinear operators has been extensively studied by many authors. After the theory of Furi, Martelli and Vignoli, a new definition was introduced by the author. Later, the work was generated to semilinear operators. In this paper, some results on the relationship between the eigenvalues and the spectrum of a positively homogeneous operators were obtained. Applying the results, we prove a theorem that gives a condition for a compact, positive operator to have a positive eigenvalue and eigenvector. The theorem can be used in the study of a second order differential equations with a three point boundary value conditions that has been studied recently by some authors.

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Positive solutions of differential equations with nonlinear boundary conditions

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Using the theory of fixed point index, we establish new results for some differential equations subject to nonlinear boundary conditions. We obtain existence of at least one or of multiple positive solutions.

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On the solvability of implicit complementarity problem and implicit variational inequalities — A unified approach and implicit projected dynamical system

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In this first part of this paper we will present a unified approach of the study of Implicit Complementary Problems and Implicit Variational Inequalities. This study is based on the concept of "Exceptional Family of Elements" for a function. This concept is obtained in this case using a kind of implicit Leray-Schauder alternative. In the second part of this paper we will present a study of solutions of Implicit Complementary Problems and Implicit Variational Inequalities, from the dynamical point of view. This study is obtained using an implicit global projected dynamical system. This paper will be finished by comments and open problems.

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Multiple positive solutions of conjugate boundary value problems with singularities

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We consider the existence of one or several nonzero positive solutions for a higher order nonlinear ordinary differential equation with conjugate boundary conditions. The conjugate boundary value problems can be changed into a Hammerstein integral equation with a suitable kernel. We shall show that the kernel has upper and lower bounds. This enables us not only to exhibit a new property of positive solutions for the conjugate boundary value problems but also to derive new results on the conjugate

boundary value problems from the well-known results on the existence of one or several positive solutions of Hammerstein integral equations with singularities obtained by the author recently. Our results generalize some known results where stronger conditions were imposed and the theory of fixed point index for compact maps defined on cones was used directly.

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Riccati equations

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We will use the Riccati equation to prove oscillation theorems for self-adjoint vector differential equations on time scales.

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An Existence Result on Positive Solutions for a Class of Semilinear Elliptic Systems

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We consider the system

$$\Delta u + \lambda f(v) = 0, x \in \Omega$$

$$\Delta v + \lambda f(u) = 0, x \in \Omega$$

$$u = 0 = v, x \in \partial\Omega$$

where λ is a positive parameter and Ω is a bounded domain. Assuming $f(x) > L, g(x) > L$ for $x > K$ for some $L > 0$ and $K > 0$, and $\lim_{x \rightarrow \infty} \frac{f(Mg(x))}{x} = 0$ for every $M > 0$, we establish the existence of a large positive solution (u, v) for λ large. In particular, we do not assume any monotonicity assumptions on f or g nor any sign conditions on $f(0)$ or $g(0)$.

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Remarks on positive solutions of some 3-point boundary value problem

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Some recent work on existence of one or of multiple solutions of a nonlinear second order differential equation with nonlocal boundary conditions will be discussed by the method of fixed point index. An optimal value will be given for a constant that appears in the definition of the cone being used and in some of the other hypotheses.

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On the existence of explosive solutions for semilinear elliptic problems

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