

Solution to Problem 5.87

Find the reactions at A and D.

First find pressures at each point where there is a change:

$$P_D = 0$$

$$P_C = \rho \cdot g \cdot y_C = 10^3 \cdot 9.81 \cdot (0.75 - 0.6)$$

$$P_C = \mathbf{1.47 \text{ kPa}}$$

$$P_B = P_C$$

$$P_A = \rho \cdot g \cdot y_A = 10^3 \cdot 9.81 \cdot 0.75 = \mathbf{7.36 \text{ kPa}}$$

Then find equivalent forces:

$$F_1 = (P_C - P_D) \cdot A_{DC} / 2 = (P_C - P_D) \cdot b \cdot (y_C - y_D) / 2 = (1.47 - 0) \cdot 1 \cdot (0.75 - 0.6) / 2 = \mathbf{110 \text{ N}}$$

$$F_2 = P_C \cdot A_{CB} = 1.47 \cdot 1 \cdot 0.6 = \mathbf{882 \text{ N}}$$

$$F_3 = (P_B + P_A) \cdot A_{DC} / 2 = (P_B + P_A) \cdot b \cdot (y_B - y_A) / 2 = (1.47 + 7.36) \cdot 1 \cdot (0.6 - 0) / 2 = \mathbf{2649 \text{ N}}$$

To find the reactions, first take moments about A. Rather than worry about the centroids of F_1 , F_2 , & F_3 , we will just find the moment arms of those forces, and the weights about point A. Denoting the perpendicular moment arm as d :

$$d_{F_1} = (0.75 - 2 \cdot (0.75 - 0.6) / 3) = 0.65 \text{ m}$$

$$d_{F_2} = 0.6 / 2 = 0.3 \text{ m}$$

$$\text{The centroid of } F_3 \text{ measured from A is: } \bar{y} = \frac{(y_B - y_A)}{3} \left(\frac{P_A + 2P_B}{P_A + P_B} \right) = 0.233 \text{ m}$$

$$\therefore d_{F_3} = 0.23 \text{ m}$$

$$\Sigma M_{ccw} = W \cdot 0.6 + W \cdot 0.3 - D_x \cdot 1.2 - F_1 \cdot 0.65 - F_2 \cdot 0.3 - F_3 \cdot 0.233 = 0 \text{ ----> } D_x = \mathbf{125 \text{ N}}$$

$$\Sigma F_x = D_x + F_1 + F_3 - A_x = 125 + 110 + 2649 - A_x = 0 \text{ ----> } A_x = \mathbf{2,884 \text{ N}}$$

$$\Sigma F_y = -3 \cdot W + F_2 + A_y = -3 \cdot 1227 + 882 + A_y = 0 \text{ ----> } A_y = \mathbf{2800 \text{ N}}$$

