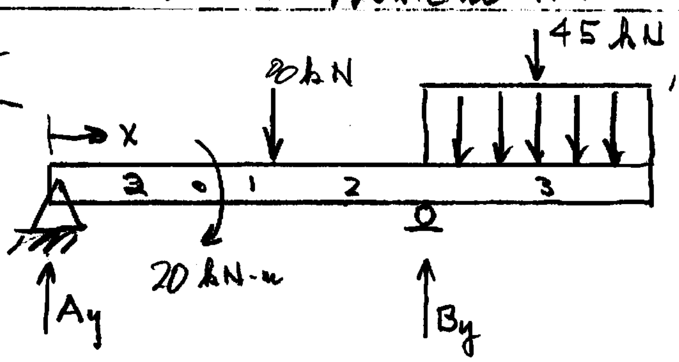
7-75



Reactions

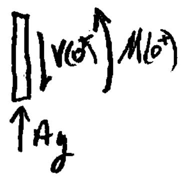
$+\circlearrowleft \Sigma M_A = 0 = B_y \cdot 5 - 20 - 8 \cdot 3 - 45 \cdot 6.5$

$B_y = 67.3 \text{ kN}$

$+\uparrow \Sigma F_y = A_y + B_y - 8 - 45 = 0$

$A_y = -14.3 \text{ kN}$

At A:



By inspection,

$M(0^+) = 0$

$+\uparrow \Sigma F_y = 0 = A_y - V(0^+) \Rightarrow V(0^+) = A_y = -14.3 \text{ kN}$

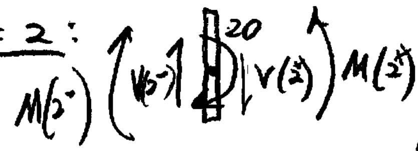
$0 \le x \le 2 \text{ m}$:

$\int_{V(0^+)}^{V(x)} dV = -\int_0^x 0 \Rightarrow V(x) = V(0^+) = -14.3 \text{ kN} \qquad V(2^-) = -14.3$

$\int_{M(0^+)}^{M(x)} dM = \int_0^x V(x)\,dx \Rightarrow M(x) - M(0^+) = V(x) \cdot x \Rightarrow M(x) = -14.3 \cdot x$

$M(2^-) = -28.6$

At $x = 2$:



$+\uparrow \Sigma F_y = 0 = V(2^-) - V(2^+) \Rightarrow V(2^+) = -14.3$

$\circlearrowleft + \Sigma M = 0 = M(2^+) - 20 - M(2^-)$

$M(2^+) = 20 + M(2^-) = 20 - 28.6 = -8.6$

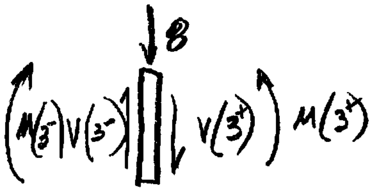
$2 < x < 3$

$\int_{V(2)}^{V(x)} dV = -\int_2^x 0 \Rightarrow V(x) = -14.3 \qquad V(3^-) = -14.3$

$\int_{M(2^+)}^{M(x)} dM = \int_2^x V(x)\,dx \Rightarrow M(x) = M(2^+) - 14.3(x-2) = 20 - 14.3x$

$M(3^-) = -22.9$

At $x = 3$:



$+\uparrow \Sigma F_y = 0 = V(3^-) - V(3) - 8$

$V(3^+) = V(3^-) - 8 = -22.3$

$\circlearrowleft + \Sigma M = 0 = M(3^+) - M(3^-)$

$M(3^+) = M(3^-) = -22.9$

$\underline{3 < x < 5}$:

$$\int_{V(3^+)}^{V(x)} dV = -\int_3^x 0\, dx \Rightarrow V(x) = V(3) = -22.3$$

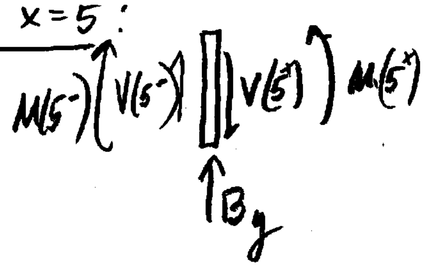
$$V(5^-) = -22.3$$

$$\int_{M(3^+)}^{M(x)} dM = \int_3^x V(x)\,dx \Rightarrow M(x) = M(3^+) - 22.3(x-3)$$

$$M(x) = 44 - 22.3x$$

$$M(3^+) = -22.9 \; ; \; M(5^-) = -67.5$$

$\underline{\text{At } x=5}$:



$$+\uparrow \Sigma F_y = 0 = V(5^-) + B_y - V(5^+)$$

$$V(5^+) = V(5^-) + B_y = -22.3 + 67.3$$

$$V(5^+) = 45$$

$$\circlearrowleft + \Sigma M = 0 = M(5^+) - M(5^-) \Rightarrow M(5^+) = -67.5$$

$\underline{5 < x \le 8}$:

$$\int_{V(5^+)}^{V(x)} dV = -\int_5^x w(x)\,dx \Rightarrow V(x) = V(5^+) - 15x\Big|_5^x$$

$$V(x) = 120 - 15x$$

$$V(5^+) = 45 \; ; \; V(8^-) = 0$$

$$\int_{M(5^+)}^{M(x)} dM = \int_5^x V(x)\,dx$$

$$M(x) = M(5^+) + \int_5^x (120 - 15x)\,dx = -67.5 + \left(120x - \frac{15}{2}x^2\right)_5^x$$

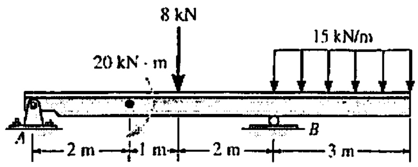
$$M(x) = -480 + 120x - 7.5x^2$$

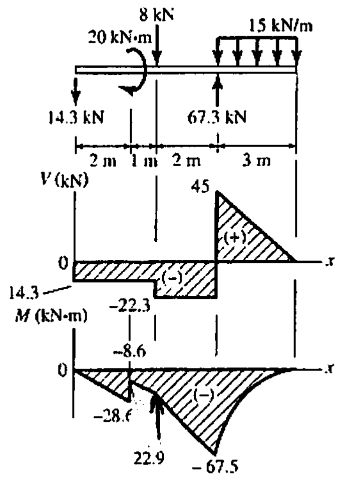
$$M(5^+) = -67.5 \; ; \; M(8^-) = 0$$

$\underline{\text{At } x=8}$; Free end. Therefore $V(8) = V(8^-) = 0$

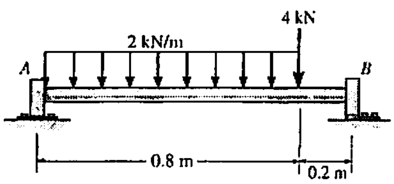
$$M(8) = M(8^-) = 0$$

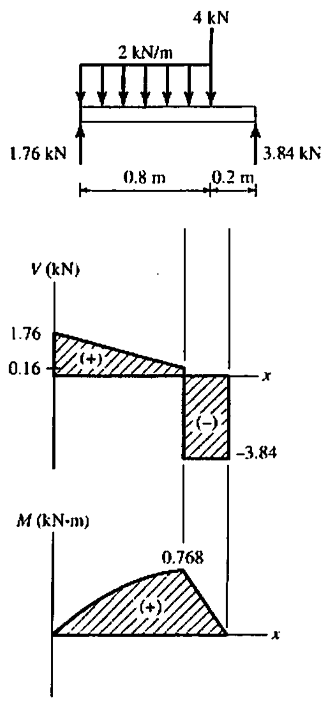
7-75. Draw the shear and moment diagrams for the beam.



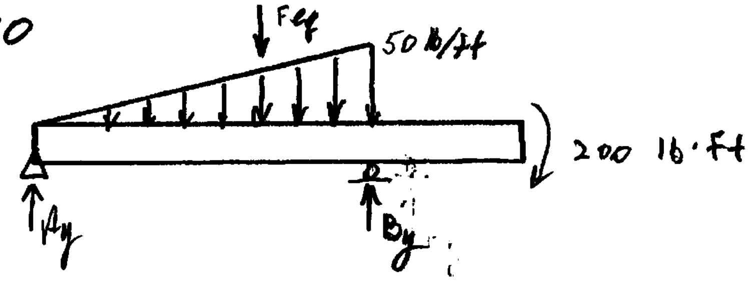


***7-76.** Draw the shear and moment diagrams for the shaft. The support at A is a thrust bearing and at B it is a journal bearing.





7.-80

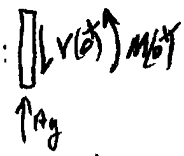


$F_{eq} = \frac{1}{2}(9)(50) = 225$

$w(x) = \frac{50-0}{9-0} = 5.56x$

Reactions; $\circlearrowleft + \Sigma M_A = 0 = B_y \cdot 9 - 200 - 6 \cdot 225 \Rightarrow B_y = 172.2$

$\uparrow \Sigma F_y = 0 = A_y + B_y - 225 = 0 \Rightarrow A_y = 52.8$

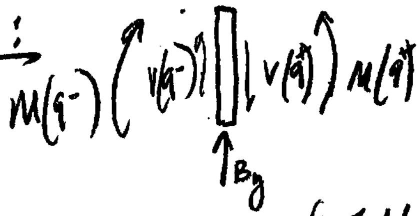
At A:  $M(0^+) = 0$; $+\uparrow \Sigma F_y = 0 = A_y - V(0^+)$; $V(0^+) = A_y = 52.8$ lb

$0 < x < 9$: $\int_{V(0^+)}^{V(x)} dV_1 = -\int_0^x w(x)\,dx$; $V(x) = V(0^+) - \int_0^x 5.56x\,dx = 52.8 - 2.78x^2$

$V(x) = 52.8 - 2.78x^2$

$\int_{M(0^+)}^{M(x)} dM_1 = \int_0^x 52.8 - 2.78x^2$; $M(x) = 52.8x - 0.93x^3$

$V(9^-) = -172.2$ lb ; $M(9^-) \doteq -200$ lb·ft

At $x = 9$:  $+\uparrow \Sigma F_y = 0 = V(9^-) + B_y - V(9^+)$

$V(9^+) = 0$ lb

$\circlearrowleft + \Sigma M = 0 = M(9^+) - M(9^-)$; $M(9) = -200$ lb·ft

$9 < x < 13$; Shear is zero & there is no change in moment. $M(13^-) = M(9) = -200$

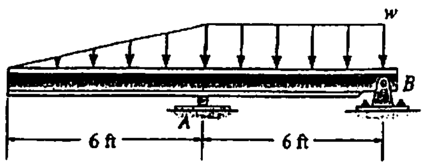
At $x = 13$:  $+\uparrow \Sigma F_y = 0 = V(13^-) - V(13^+) \Rightarrow V(13) = 0$

$+\circlearrowleft \Sigma M = 0 = M(13^+) - M(13^-)$

$M(13^+) = M(13^-)$

$-200 = -200$

7-78. The beam will fail when the maximum moment is $M_{max} = 30$ kip · ft or the maximum shear is $V_{max} = 8$ kip. Determine the largest distributed load w the beam will support.



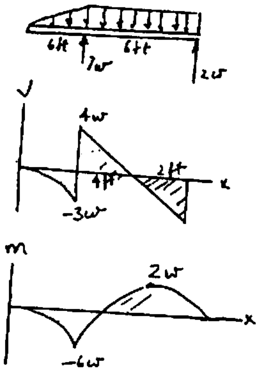
$V_{max} = 4w; \quad 8 = 4w$

$w = 2$ kip/ft

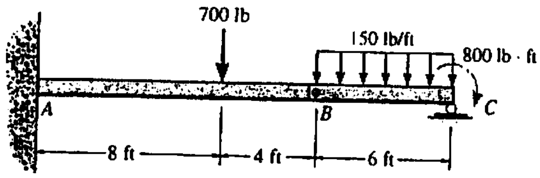
$M_{max} = -6w; \quad -30 = -6w$

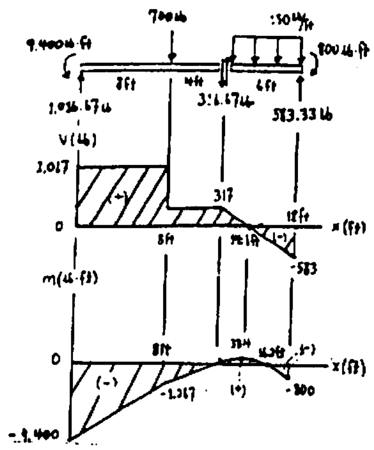
$w = 5$ kip/ft

Thus, $w = 2$ kip/ft **Ans**

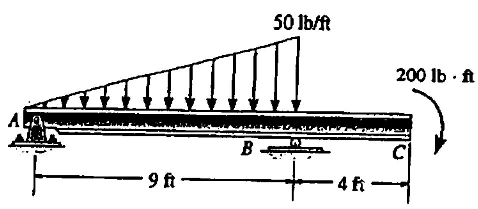


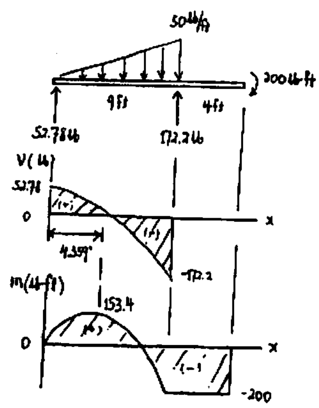
7-79. The beam consists of two segments pin connected at B. Draw the shear and moment diagrams for the beam.





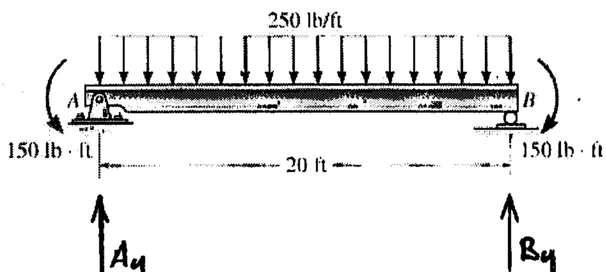
***7-80.** Draw the shear and moment diagrams for the beam.





Integral Method applied to Problem 7.51.

By inspection, $A_y = B_y = \dfrac{250\,\frac{lb}{ft}(20)}{2} = 2500\,lb$



At A: $150 \; V(0^+) \; M(0^+)$ $\boxed{x=0}$ A_y

$+\uparrow \Sigma F_y = 0 = A_y - V(0^+) \Rightarrow V(0^+) = A_y$

$\circlearrowleft + \Sigma M_A = 0 = M(0^+) + 150 \Rightarrow M(0^+) = -150$

$w(x) = \text{const.} = w$

$\underline{0 \le x < 20}$: $\dfrac{dV(x)}{dx} = -w(x) = -w$

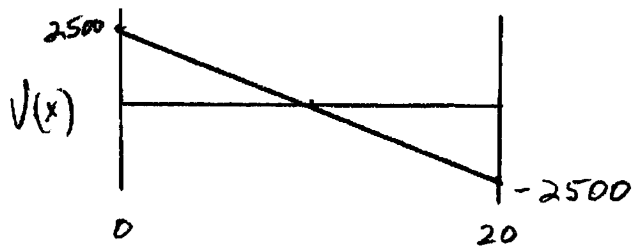
$$\int_{V(0^+)}^{V(x)} dV = -w\int_0^x dx$$

$V(x) - V(0^+) = -w(x-0)$

$V(x) - A_y = -wx$

$V(x) = A_y - wx = 2500 - 250x$

$V(0) = 2500;\ V(20^-) = -2500\,lb.$



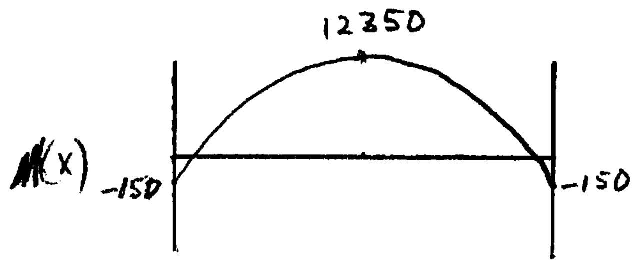
At B $x=20$

$V(20^-)\uparrow \; \downarrow V(20)$ $\quad B_y$ $\quad \uparrow\Sigma F_y = 0 = B_y + V(20^-) - V(20)$

$V(20) = B_y + V(20^-) = 2500 - 2500 = 0$

$$\int_{-150}^{M(x)} dM = \int_0^x V(x)\,dx$$

$M(x) - (-150) = \int_0^x (A_y - wx)\,dx = A_y x - \dfrac{wx^2}{2}$



$M(x) = -150 + 2500x - 125x^2$

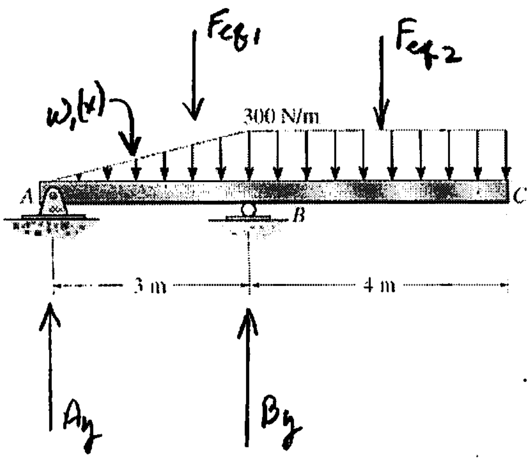
$M(0) = -150\ lb\cdot ft$

$M(20^-) = -150\ lb\cdot ft$

$M(20^-) \; M(20)$ $\quad \circlearrowleft + \Sigma M = 0 = M(20) - M(20^-)$

$M(20) = M(20^-) = -150\ lb\cdot ft$

Integral Method applied to Problem 7.60



$F_{eq_1} = \frac{1}{2}(300)\,3 = 450\text{ N}$

$w_1(x) = 100\cdot x \text{ N/m}$

$F_{eq_2} = 300\cdot 4 = 1200\text{ N}$

$w_2 = 300 \text{ N/m}$

$\circlearrowleft + \Sigma M_A = 0 = B_y\cdot 3 - 450\cdot 2 - 1200\cdot 5$

$\boxed{B_y = 2300}$

$\uparrow + \Sigma F_y = 0 = A_y + B_y - 450 - 1200 \quad \boxed{A_y = -650}$

At A:

$V(0^+) = A_y;\ M(0^+) = 0.$

$\uparrow A_y$

$0 < x < 3$

$$\int_{V(0^+)}^{V(x)} dV = \int_0^x -100\cdot x\, dx$$

$V(x) - V(0^+) = -100\frac{x^2}{2} \Rightarrow V(x) = A_y - 50x^2$

$V(x) = -650 - 50x^2$

$V(0^+) = -650;\ V(3^-) = -1100.$

$$\int_{M(0^+)}^{M(x)} dM = \int_0^x V(x)\,dx = \int_0^x (-650 - 50x^2)\,dx$$

$M(x) - \cancel{M(0^+)}^{\,0} = -650x - 50\frac{x^3}{3}$

$M(0^+) = 0;\ M(3^-) = -2400.$

At B:

$M(3^-)\ V(3^-)\ V(3^+)\ M(3^+)$

$\uparrow B_y$

$+\uparrow \Sigma F_y = 0 = V(3^-) + B_y - V(3^+);\quad V(3^+) = V(3^-) + B_y = 1200\text{ N}$

$\circlearrowright \Sigma M = 0 = M(3^+) - M(3^-);\quad M(3^+) = M(3^-) = -2400\text{ N}\cdot\text{m}$

$3 \le x < 7$

$$\int_{V(3)}^{V(x)} dV = \int_3^x -300\cdot dx$$

$V(x) - V(3) = -300(x-3)$

$V(x) = 1200 - 300\cdot x + 900$

$V(3) = 1200\text{ N};\ V(7^-) = 0.$

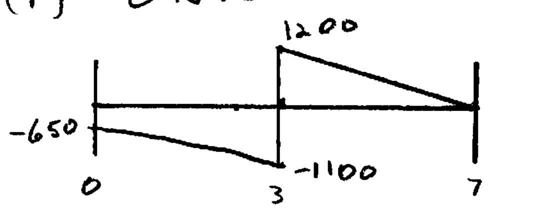
$$\int_{M(3)}^{M(x)} dM = \int_3^x (2100 - 300\cdot x)\,dx$$

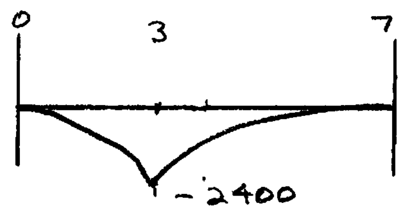
$M(x) - M(3) = 2100(x-3) - 150(x^2 - 9)$

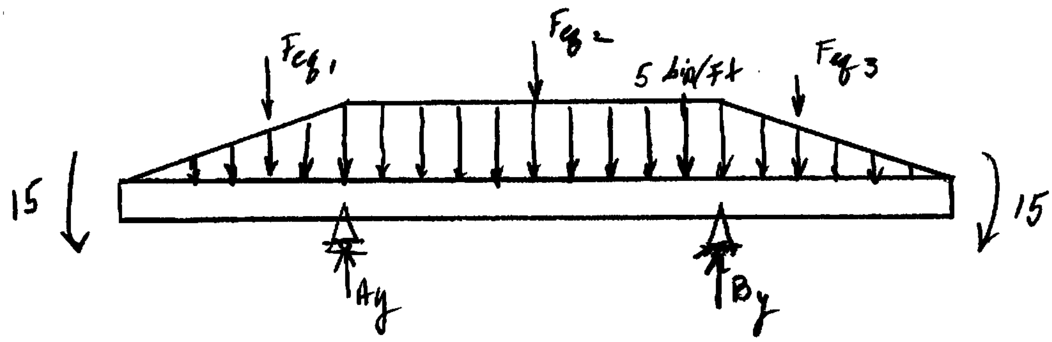
$M(x) = -150x^2 + 2100x - 7350$

$M(3) = -2400\text{ N}\cdot\text{m}$

$M(7^-) = 0\text{ N}\cdot\text{m}$







$F_{eq1} = F_{eq3} = \frac{1}{2}(6)(5) = 15$ kips $\quad F_{eq2} = 50$ kips

$w(x)_{left} = \frac{5-0}{6-0}x = \frac{5}{6}x$; $\quad w(x)_{right} = \frac{0-5}{22-16}x + \text{Intercept} = \frac{-5}{6}x + \text{Int.}$

$w(16) = 5$; $\quad 5 = -\frac{5}{6}(16) + \text{Int.}$

$\text{Int.} = 18.33$; $\boxed{w(x)_{right} = \frac{-5}{6}x + 18.33}$

Reactions:

$+\circlearrowleft \Sigma M_A = 0 = F_{eq1}\cdot 2 + B_y \cdot 10 - F_{eq2}\cdot 5 - F_{eq3}\cdot 12 + 15 - 15 \Rightarrow B_y = 40$

$+\uparrow \Sigma F_y = 0 = A_y + B_y - 15 - 15 - 50 \Rightarrow A_y = 40$

Or by symmetry $A_y = B_y = \frac{1}{2}(15+15+50) = 40.$

At A:

$+\uparrow \Sigma F_y = 0 = -V(0^+) = 0 \quad V(0^+) = 0$

$+\circlearrowleft \Sigma M = 0 = M(0^+) + 15 \Rightarrow M(0^+) = -15$ kip-ft

$0 < x \le 6$:

$\int_{V(0^+)=0}^{V(x)} dV(x) = -\int_0^x w(x)_{left}\, dx$; $\quad V(x) = \int_0^x -\frac{5}{6}x\,dx = -\frac{5}{12}x^2$

$V(6^-) = -15$

$\int_{M(0^+)}^{M(x)} dM(x) = \int_0^x -\frac{5}{12}x^2\,dx$; $\quad M(x) = (-15) - \frac{5}{36}x^3 = -15 - \frac{5}{36}x^3$

$M(0^+) = -15$; $\quad M(6^-) = -45.$

At 6:

$+\uparrow \Sigma F_y = 0 = V(6^-) + A_y - V(6^+)$

$V(6^+) = -15 + 40 = 25$

$+\circlearrowleft \Sigma M = 0 = M(6^+) - M(6^-) \quad M(6^+) = M(6^-) = -45$

$6 \le x < 16$

$$\int_{V(6^+)}^{V(x)} dV = -\int_6^x 5\,dx \;; \quad V(x) = V(6^+) - 5(x-6)$$

$$V(x) = 25 - 5x + 30 = 55 - 5x$$

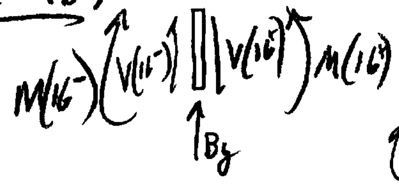
$$V(6^+) = -25,\; V(16^-) = -25$$

$$\int_{M(6)}^{M(x)} dM = \int_6^x 55 - 5x \;; \quad M(x) = M(6^+) + 55(x-6) - \frac{5}{2}(x^2 - 36)$$

$$M(x) = -285 + 55x - 2.5x^2$$

$$M(6^+) = -45 \;; \quad M(16^-) = -45$$

At $x = 16$:



$$+\uparrow \Sigma F_y = 0 = V(16^-) + B_y - V(16^+)$$

$$V(16^+) = -25 + 40 = 15$$

$$\circlearrowleft + \Sigma M = 0 = M(16^+) - M(16^-) \;; \quad M(16^+) = -45$$

$16 \le x < 22$

$$\int_{V(16^+)}^{V(x)} dV = -\int_{16}^x w(x)_{right}\,dx = -\int_{16}^x -\frac{5}{6}x + 18.33$$

$$V(x) = 15 + \frac{5}{12}(x^2 - 256) - 18.33(x - 16)$$

$$V(x) = 201.61 - 18.33x + \frac{5}{12}x^2$$

$$V(16^+) = 15 \;; \quad V(22^-) = 0.$$

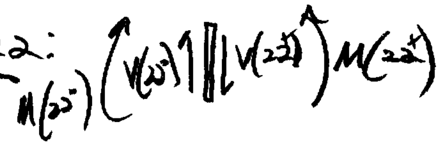
$$\int_{M(16^+)}^{M(x)} dM = \int_{16}^x 201.61 - 18.33x + \frac{5}{12}x^2 = 201.61(x-16) - \frac{18.33}{2}(x^2 - 16^2) + \frac{5}{36}(x^3 - 16^3)$$

$$M(x) = -45 + (201.61x - 3225.76) - \frac{18.33}{2}(x^2 - 256) + \frac{5}{36}(x^3 - 4096)$$

$$M(x) = -1493.41 + 201.61x - \frac{18.33}{2}x^2 + \frac{5}{36}x^3$$

$$M(16^+) = -45 \;; \quad M(22^-) = -15$$

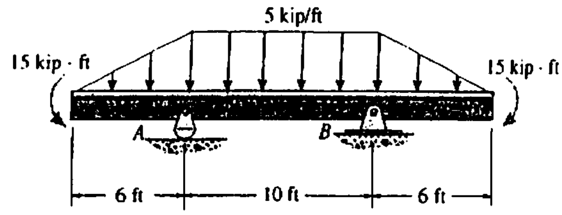
At $x = 22$:



$$+\uparrow \Sigma F_y = 0 = V(22^-) - V(22^+) \;; \quad V(22^+) = 0$$

$$\circlearrowleft + \Sigma M = 0 = M(22^+) - M(22^-) \;; \quad M(22^+) = -15$$

7-86. Draw the shear and moment diagrams for the beam.



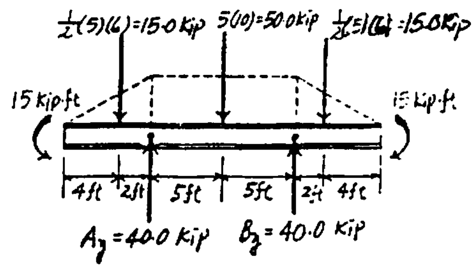
Support Reactions : From FBD (a).

$$\circlearrowleft + \Sigma M_A = 0; \quad B_y(10) + 15.0(2) + 15 - 50.0(5) - 15.0(12) - 15 = 0$$

$$B_y = 40.0 \text{ kip}$$

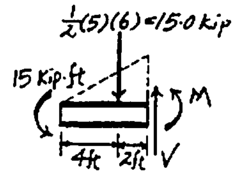
$$+\uparrow \Sigma F_y = 0; \quad A_y + 40.0 - 15.0 - 50.0 - 15.0 = 0$$

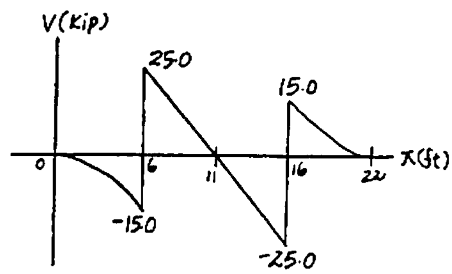
$$A_y = 40.0 \text{ kip}$$

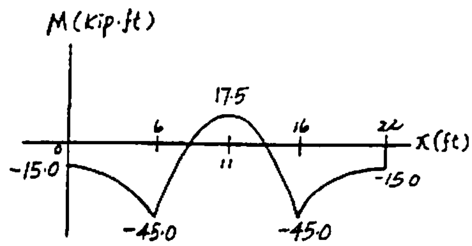


Shear and Moment Diagrams : The value of the moment at supports A and B can be evaluated using the method of sections [FBD (c)].

$$+\Sigma M = 0; \quad M + 15.0(2) + 15 = 0 \quad M = -45.0 \text{ kip} \cdot \text{ft}$$







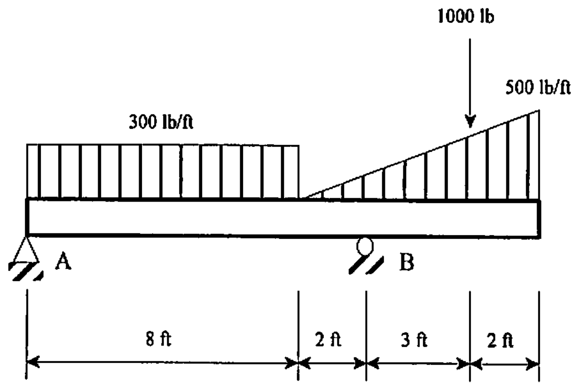
MAE 206: Statics

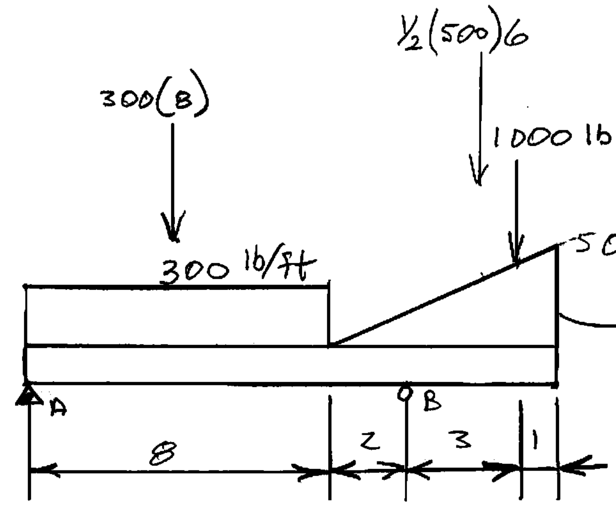
Test 5 - take home

Name: ______________________________

Open book, notes. Be sure to include appropriate units and signs in your answers. Drawing must be neat and readable. Do not leave out steps. Clearly place coordinate systems if different from that give in the figures.

Problem 1. Draw the shear and moment diagrams for the beam shown using the integral method.





$w = \frac{500-0}{14-8} = \frac{500}{6}$

$w(x) = \frac{500}{6}x + I$

$w(8) = 0$

$0 = \frac{500(8)}{6} + I \Rightarrow I = -\frac{2000}{3}$

$w(x) = \frac{500}{6}x - \frac{2000}{3}$

Reactions:

$\circlearrowleft + \Sigma M_A = 0 = R_B \cdot 10 - 2400 \cdot 4 - \frac{3000}{2} \cdot 12 - 10^3 \cdot 13 \Rightarrow \boxed{R_B = 4060 \text{ lb}}$

$\uparrow + \Sigma F_y = 0 = R_A + R_B - 2400 - 1500 - 1000 \Rightarrow \boxed{R_A = 840 \text{ lb}}$

At A:

$V(0^+)$, $M(0^+)$, R_A

$\uparrow \Sigma F_y = 0 = R_A - V(0^+) \Rightarrow V(0^+) = R_A = 840 \text{ lb}$

$\circlearrowleft + \Sigma M = 0 = M(0^+) = 0 \Rightarrow M(0^+) = 0$

$0 < x < 8$

$w(x) = 300 \frac{\text{lb}}{\text{ft}}$

$\int_{V(0^+)}^{V(x)} dV = -\int_0^x w(x)\,dx \qquad V(x) - V(0^+) = -300x$

$V(x) = -300x + 840 \qquad V(0^+) = 840 ✓ \quad V(8) = -1560$

$\int_{M(0^+)}^{M(x)} dM = \int_0^x V(x)\,dx = \int_0^x -300x + 840$

$M(x) - \cancel{M(0^+)}^{0} = -\frac{300}{2}x^2 + 840x \Big|_0^x \Rightarrow M(x) = -150x^2 + 840x$

$M(0^+) = 0 ✓ \quad M(8^-) = -2880$

At $x = 8$:

$M(8^-)$, $V(8^-)$, $V(8^+)$, $M(8^+)$

$\uparrow \Sigma F_y = 0 = V(8^-) - V(8^+) \Rightarrow V(8^+) = V(8^-)$

$\circlearrowleft + \Sigma M = 0 = M(8^+) - M(8^-) \Rightarrow M(8^+) = M(8^-)$

$8 < x < 10$:

$\int_{V(8^+)}^{V(x)} dV = \int_8^x -w(x)\,dx = \int_8^x -\left(\frac{500}{6}x - \frac{2000}{3}\right)dx$

$V(x) - (-1560) = -\frac{500}{12}x^2 + \frac{2000}{3}x \Big|_8^x \qquad V(8^+) = -1560 ✓$

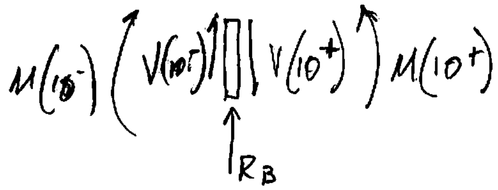
$V(x) = -41.7x^2 + 666.7x - 4{,}226.7 \qquad V(10^-) = -1726.7$

$$\int_{M(8^+)}^{M(x)} dM = \int_8^x V(x)\,dx = \int_8^x \left(-41.7x^2 + 666.7x - 4{,}226.7\right) dx$$

$$M(x) = -13.9x^3 + 333.3x^2 - 4{,}226.7x + 16{,}711$$

$$M(8^+) = -2{,}886 \checkmark \qquad M(10^-) = -6{,}111$$

<u>At $x = 10$:</u>



$$+\uparrow \Sigma F_y = 0 = V(10^-) + R_B - V(10^+)$$

$$V(10^+) = V(10^-) + R_B = 2{,}333.3$$

$$\circlearrowleft \Sigma M = 0 = M(10^+) - M(10^-) \qquad M(10^+) = -6{,}111$$

<u>$10 < x < 13$:</u>

$$\int_{V(10^+)}^{V(x)} dV = -\int_{10}^x w(x)\,dx = \int_{10}^x -\left(\frac{500x}{6} - \frac{2000}{3}\right) dx$$

$$V(x) = V(10^+) - \left(\frac{500}{12}x^2 - \frac{2000}{3}x\right)\Big|_{10}^x$$

$$V(x) = -41.7x^2 + 666.7x - 166.7$$

$$V(10^+) = 2333 \checkmark \qquad V(13^-) = 1{,}458.3$$

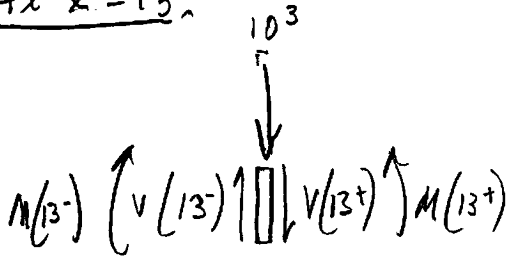
$$\int_{M(10^+)}^{M(x)} dM = \int_{10}^x V(x)\,dx = \int_{10}^x \left(-41.7x^2 + 666.7x - 166.7\right) -$$

$$M(x) = M(10^+) + \left(-13.9x^3 + 333.3x^2 - 166.7x\right)\Big|_{10}^x$$

$$M(x) = -13.9x^3 + 333.3x^2 - 166.7x + 23{,}888.8 \qquad M(10^+) = -6111 \checkmark$$

$$M(13^-) = -236$$

<u>At $x = 13$:</u>



$$+\uparrow \Sigma F_y = 0 = V(13^-) - 10^3 - V(13^+)$$

$$V(13^+) = 458.3$$

$$\circlearrowleft \Sigma M = 0 = M(13^+) - M(13^-)$$

$$M(13^+) = -236$$

$\underline{13 < x < 14}$:

$$\int_{V(13^+)}^{V(x)} dV = \int_{13}^{x} -w(x)\,dx = -\int_{13}^{x}\left(\frac{500x}{6} - \frac{2000}{3}\right)dx$$

$$V(x) = V(13^+) - \left(41.7x^2 - 666.7x\right)\Big|_{13}^{x}$$

$$V(x) = -41.7x^2 + 666.7x - 1{,}166.7 \qquad V(13^+) = 458.3 \checkmark$$

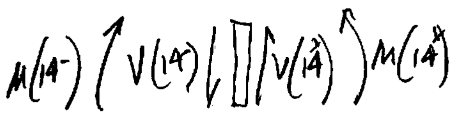
$$V(14^-) = 0$$

$$\int_{M(13^+)}^{M(x)} dM = \int_{13}^{x}\left(-41.7x^2 + 666.7x - 1166.7\right)dx$$

$$M(x) = -13.9x^3 + 333.3x^2 - 1166.7x - 10{,}888.8$$

$$M(13^+) = -236 \checkmark \quad M(14^-) = 0$$

$\underline{\text{At } x = 14}$



$$+\!\uparrow \Sigma F_y = 0 = V(14^+) - V(14^-) = 0 \qquad V(14) = 0 \checkmark$$

$$\circlearrowleft\!+ \Sigma M = 0 = M(14^+) - M(14^-) = 0 \qquad M(14) = 0 \checkmark$$